

Algebra II

Chapter 12.2: Conditional probability

Objective: Find probability of one event given another has occurred

Conditional Probability: ^{Dependent} Given something has occurred

For any two events A and B, the probability of event B given that A has occurred is

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ ← Both
← first

Must be dependent events; Table and tree diagrams can help

Rearranged formula: $P(A \text{ and } B) = P(A) (P(B|A))$ → dependent event formula

Reading tabular data. no formula needed

Ex1. A student committee is being formed to decide how after-school activities will be funded. The committee members are selected at random from current club members. The frequency table shows the current club membership data.

What is the probability that a member of the art club selected at random is a junior? ^{already happened} ↓ now looking

What is the condition? →

Monday Club Memberships by Grade

	Drama	Science	Art	Total
Sophomore	3	9	24	36
Junior	6	18	16	40
Senior	8	13	18	39
Total	17	40	58	115

$P(J|A)$
Look at Art column total 58
 $P(J|A) = \frac{16}{58} = \frac{8}{29}$

What is the probability that a member of the drama club is a sophomore?

given

look at Drama total = 17

$\rightarrow \frac{3}{17} = P(S|D)$

What is the probability that a sophomore is a member of the drama club, (drama||sophomore)? Is (sophomore||drama) the same as P(drama||sophomore)? Explain.

look at sophomore total = 36

$P(D|S) = \frac{3}{36} = \frac{1}{12}$ not the same

Conditional probability and independent events:

IF $P(B|A) = P(B)$ and $P(A|B) = P(A)$ then the events are independent

Use the test for independence:

Ex2. The table below shows the vehicles in a parking garage one afternoon. A vehicle in the garage will be selected at random. Let B represent "the vehicle is black" and V represent "the vehicle is a van." Are the events B and V independent or dependent?

	Car	Van	Pickup	Totals
Red	5	0	2	7
White	0	0	2	2
Black	6	3	4	13
Totals	11	3	8	22

$P(B|V) = P(B)$?
 $P(B|V) = \frac{3}{3} = 1$
 $P(B) = \frac{3}{22}$ not same
 so dependent

You try.

Probability from tables

A utility company asked 50 of its customers whether they pay their bills online or by mail. What is the probability that a customer pays the bill online given that the customer is male?

What is the condition?

male
total 20

$P(O|M) = 12/20 = \frac{3}{5} \approx 60\%$

	Online	By Mail
Male	12	8
Female	24	6

The table shows students by gender at two and four year colleges and graduate schools. You pick a student at random. What is the $P(\text{female}|\text{graduate student})$? What is $P(4 \text{ year}|\text{male})$?

	Males (in thousands)	Females (in thousands)
Two-year colleges	1866	2462
Four-year colleges	4324	5517
Graduate schools	1349	1954

4324
 9841
 3303

 7539 | 9933 | 17472

$P(F|G) = \frac{1954}{3303} \approx 59\%$

$P(4|M) = \frac{4324}{7539} \approx 57\%$

Total G = 3303

total m = 7539

Applying the formula.

Ex3. A band's marketing agent conducted a survey to determine how many high school fans the band has. What is the probability that a surveyed student plans to attend the band's concert and is a fan of the group?

What is the probability that a surveyed student plans to attend but is not a fan of the group?

$P(A) = \text{attend}$ $P(F) = \text{Fan}$
 $P(A \text{ and } F) = P(A) \cdot P(F|A)$
 $= .70 (.8) = .56 \text{ } 56\%$


Students who plan to attend concert	<ul style="list-style-type: none"> 70% of students plan to attend, 80% of students who plan to attend are fans of the band.
Students who do not plan to attend	<ul style="list-style-type: none"> 30% of students do not plan to attend, 25% are fans of the band.

You try. Eighty percent of an airline's flights depart on schedule. Seventy-two percent of its flights depart and arrive on schedule. Find the probability that a flight that departs on time also arrives on time.

What is the condition? *departs* $P(A|D) = \frac{P(A \text{ and } D)}{P(D)}$ *Given departs*

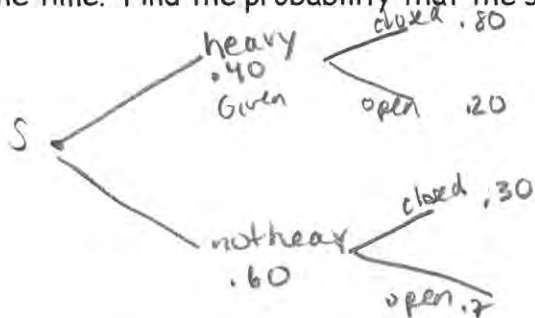
$$= \frac{.72}{.8} = .9 \text{ or } 90\%$$

Ex4. A marketer is looking at mobile phone statistics to help plan an online advertising campaign. She wants to find which product is most likely to be purchased after a related search.

Mobile Phone Search and Buying Behavior 		
Product	Search(S)	Search & Buy (S and B)
W	46%	16%
X	32%	14%
Y	35%	12%
Z	40%	15%

Tree diagram

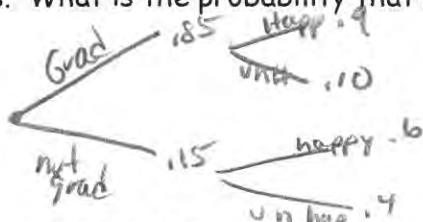
Ex5 Make a tree diagram based on the observations below: Of all snowfalls, 40% are heavy. After a heavy snowfall, schools are closed 80% of the time. After a light snowfall, schools are closed 30% of the time. Find the probability that the schools are open, given heavy snow.



$$P(O|H) = .20 \quad 20\%$$

$$P(O|L) = .7$$

You try. Use a tree diagram. A school system compiled the following information on people who were juniors ten years earlier. 85% of students graduated from high school, of the students who graduated 90% are happy with their present jobs. Of the students who did not graduate, 60% are happy with their present jobs. What is the probability that a person graduated and is happy with their current job?



$$P(G \text{ and } H) = P(G) \cdot P(H|G)$$

$$= .85 \cdot (.9)$$

$$= .765$$

$$76.5\%$$

What is the probability that a person is happy with their current job?

$$P(H) = P(H|G) \text{ or } P(H|NG)$$

$$.765 + .6(.15) = 85.5\%$$

What is the probability that a person who is happy graduated?

$$P(G|H) = \frac{P(G \text{ and } H)}{P(H)} = \frac{.765}{.855}$$

$$= .895$$

$$89.5\%$$

